

REMARK ON GROUPS OF ITERATIVE ROOTS OF THE FORMAL SERIES $F(x) = x$

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It is known that the iterative roots of the formal series $F(x) = x$ (in the group Γ of invertible series) are exactly the series $T^{-1}(\rho Tx)$, where $T \in \Gamma$ and ρ is a complex root of unity. Denote by E_Γ this set of iterative roots, lying in Γ .

Then one can show:

- (i) E_Γ is neither a subgroup of Γ nor a family of commuting series.
- (ii) Each subgroup Γ_0 of Γ , lying in E_Γ , is of the form

$$\Gamma_0 = \{T^{-1}(\rho Tx) \mid \rho \in E_0\},$$

where E_0 is a subgroup of the group E of complex roots of 1, uniquely determined by Γ_0 . If Γ_0 is infinite, then also T is uniquely determined by Γ_0 , otherwise there is a continuum of possible T for the representation.

- (iii) Hence each subgroup of Γ in E_Γ is abelian, isomorphic to a group of complex roots of unity, and hence cyclic if it is finite.

Furthermore, maximal subgroups of Γ in E_Γ exist and may be investigated in detail, using the above results, the theory of normal forms and of families of commuting series.