REMARK ON "WEAK DYNAMICS"

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Gy. Targonski mentioned in his talk once more the concept of weak dynamics, which means to consider an embedding problem where the translation equation is replaced by the system of Aczél–Jabotinsky equations or by one of those equations. This remark is intended to point out that this problem has been solved for formal power series transformations in one indeterminate and the translation equation replaced by a (weaker) Aczél–Jabotinsky differential equation of third type.

To be more precise: Let $F(x) = \rho x + c_2 x^2 + \cdots$, with $\rho \neq 0$. Then there exists a formal series $G(x) = x^m + d_{m+1}x^{m+1} + \cdots$ such that $F(x)$ is a solution of the equation

$$(G \circ \Phi)(x) = \frac{d\Phi}{dx} \cdot G(x). \tag{AJ}$$

If $F(x)$ is not of the form $F(x) = T^{-1}(\rho T x)$ with a root ρ of unity then G is uniquely determined by F.

In [1] we constructed G explicitly. If we are only interested in existence (and uniqueness) then the proof is much shorter:

1. Each $F(x) = \rho x + c_2 x^2 + \ldots$ is contained in at least one maximal family of commuting series $\mathcal{F}$. (Apply Zorn's lemma to get $\mathcal{F}$). $\mathcal{F}$ is uniquely determined if $F(x)$ is not of the form $T^{-1}(\rho T x)$ with a root ρ of unity.
2. Each maximal family $\mathcal{F}$ of commuting series is the set of all solutions of exactly one Aczél–Jabotinsky equation (AJ). (For a proof see [2]).

If the multiplier ρ of F is not a root of unity then the "weak embedding" problem is nothing else than embedding F into a 1-parameter group. But otherwise it is a new possibility.

[1] L. Reich, On the embedding problem for formal power series with respect to the Aczél–Jabotinsky equations, in: Ch. Mira et al., eds., *European Conference on Iteration Theory 1989*, World Scientific, Singapore, 1991, 294–304.

[2] L. Reich, On families of commuting formal power series, *Berichte der Mathematisch-statistischen Sektion*, Forschungsgesellschaft Joanneum Graz, Nr. 294 (1989), 1–18.