Abstracts
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SOME APPLICATIONS OF THE PERIODIC STRUCTURE OF PERMUTATION-PRODUCT MAPS

Antonio Linero[1]

The aim of this talk is to present some results recently obtained concerning he periodic structure of permutation-product maps and apply them to get for $k \geq 2$ the periodic structure of *delayed difference equations* of the form

$$x_n = f(x_{n+k})$$

on the unit interval $I = [0, 1]$ and the unit circle $\mathbb{S}^1$.

Given a compact metric space X we say that $F : X^k \to X^k$ is a *permutation-product map* if

$$F(x_1, x_2, ..., x_k) = \left(f_{\sigma(1)}(x_{\sigma(1)}), f_{\sigma(2)}(x_{\sigma(2)}), ..., f_{\sigma(k)}(x_{\sigma(k)})\right),$$

where σ is a cyclic permutation of $\{1, 2, ..., k\}$ and each $f_i : X \to X$ is continuous, $i = 1, 2, ..., k$.

In the case of $X = I$ it is possible to find a periodic structure for this class of maps, similar to that given by Šarkovskiĭ's Theorem. For $X = \mathbb{S}^1$ the periodic structure obtained for permutation-product maps is strongly related to the periodic structure in the circle. In the general case of a compact metric space X (even a simple set X), we can also obtain information of the periodic structure of F if we know what is the periodic structure in X.

Now we apply the above results on periodicity to get the periodic structure of difference equations of the form $x_{n+k} = f(x_n)$, where f is a continuous map defined from I (or $\mathbb{S}^1$) into itself, with $n = 1, 2, ...$, and $k \in \mathbb{N}$ fix. The basic idea is to define from f a particular permutation-map F, namely

$$F(x_1, x_2, ..., x_k) = (x_2, x_3, ..., x_k, f(x_1)).$$

Then the periodic structure of the delayed difference equation is given in terms of that of F.

Finally, it can be noticed that the knowledge of the periodic structure of a difference equation allows us to deduce some properties on its dynamics. For instance, we will present some results related to the existence of global attractors of $x_{n+k} = f(x_n)$.

[1]Universidad de Murcia, Spain.