

Coordinate Type	γ	Δ	a	b	$\tan(\theta)$	k^2	Singular Points
Cartesian	0	0	arbitrary	arbitrary	$\begin{cases} 0, & \text{if } C = 0, \\ \frac{B - A + \sqrt{(B - A)^2 + 4C^2}}{2C}, & \text{if } C \neq 0. \end{cases}$ <small>(θ unique mod $\pi/2$)</small>	0/0	two at ∞
Parabolic	0	$\neq 0$	$\frac{\beta(A - B) + 2\alpha C}{2(\alpha^2 + \beta^2)}$	$\frac{\alpha(B - A) + 2\beta C}{2(\alpha^2 + \beta^2)}$	$\begin{cases} \pm\infty, & \text{if } \beta = 0, \\ \alpha/\beta, & \text{if } \beta \neq 0. \end{cases}$ <small>(θ unique mod π)</small>	∞	one at \mathbf{a} , one at ∞
Polar	$\neq 0$	0	$-\frac{\beta}{\gamma}$	$-\frac{\alpha}{\gamma}$	arbitrary	0	two coincide at \mathbf{a}
Elliptic-Hyperbolic	$\neq 0$	$\neq 0$	$-\frac{\beta}{\gamma}$	$-\frac{\alpha}{\gamma}$	Define $\sigma = \alpha^2 - \beta^2 + \gamma(B - A)$. $\begin{cases} 0, & \text{if } \gamma C + \alpha\beta = 0, \sigma < 0, \\ \pm\infty, & \text{if } \gamma C + \alpha\beta = 0, \sigma > 0, \\ \frac{\sigma + \sqrt{\Delta}}{2(\gamma C + \alpha\beta)}, & \text{if } \gamma C + \alpha\beta \neq 0. \end{cases}$ <small>(θ unique mod π)</small>	$\frac{\sqrt{\Delta}}{\gamma^2}$	two distinct

Table 1: Formulae for transformation to separable coordinates

Transformation to separable coordinates:

$$\begin{pmatrix} q^1 \\ q^2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} T_\Lambda \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

Standard coordinate transformations:

$$\begin{aligned} T_C &: x = u, y = v \\ T_P &: x = u \cos v, y = u \sin v \\ T_{PB} &: x = \frac{1}{2}(u^2 - v^2), y = uv \\ T_{EH} &: x = k \cosh u \cos v, y = k \sinh u \sin v \end{aligned}$$