Czech-Slovak Workshop on
Discrete Dynamical Systems
2016

12–16 September 2016

Karlová Studánka
Czech Republic

Organized by
Mathematical Institute in Opava
Silesian University in Opava
Welcome

This year the traditional Workshop is organized in Mountain Spa Karlova Studánka in Jesenky Mountains.

Topical informations can be found on http://conferences.math.slu.cz/CSWDDS/.

Scientific programme starts on Monday at 9:00 a.m. and it ends on Friday before lunch. Each day there is a Morning session, and only on Tuesday there is also an Evening session. The rest of day can be devoted to discussions and work in groups, or individual work.

On Wednesday, instead of dinner, from 6 p.m. there is a banquet for participants of the Workshop.

Hotel information.
Phone number: +420 554 798 111
  8:00-9:00   breakfast
  11:30-12:30  lunch
  17:30-18:30  dinner
List of speakers

Balibrea, Francisco ........................................... 7  Pietrzyk, Marta ........................................... 11
Gröger, Maik ......................................................... 8  Pravec, Vojtěch ........................................... 11
Kováč, Jozef ......................................................... 8  Raith, Peter ........................................... 12
Kulczycki, Marcin ............................................... 8  Roth, Samuel ........................................... 12
Kupka, Jiří ........................................................... 9  Roth, Zuzana ........................................... 12
Kupsa, Michal ...................................................... 9  Snoha, Lubomír ........................................... 13
Lącka, Martha ...................................................... 9  Šotola, Jakub ........................................... 13
Lampart, Marek .................................................... 10  Štefánková, Marta ..................................... 14
Maličký, Peter ...................................................... 10  Takács, Michal ........................................... 14
Programme

Monday, 12 September

Morning session

Chair: Ľubomír Snoha

9:00–9:45  Francisco BALIBREA
          On the structure of forbidden sets of rational difference equations

9:45–10:00  Jozef KOVÁČ
            Random dynamical systems generated by two Allee maps

10:00–10:30  Coffee break

10:30–10:50  Marta ŠTEFÁNKOVÁ
              On generic and dense chaos for maps induced on hyper-spaces

10:55–11:15  Vojtěch PRAVEC
              On dynamics of triangular maps of the square with zero topological entropy
Tuesday, 13 September

Morning session

Chair: Francisco Balibrea

9:00–9:30 Peter MALIČKÝ
Periodic points of Lotka-Volterra map and their relation to number theory

9:30–10:00 Michal TAKÁCS
Generic chaos on graphs

10:00–10:30 Coffee break

10:30–11:15 L'ubomír SNOHA
Loops of transitive interval maps

Evening session

Chair: Marta Štefánková

19:00–19:20 Marek LAMPART
Chaotic sub-dynamics in coupled logistic maps

19:25–19:45 Jiří KUPKA
On mixing completely scrambled systems
Wednesday, 14 September

Morning session

Chair: Peter Maličký

9:00–9:30  Marcin KULCZYCKI  
_Further results on the dynamics of spacing shifts_

9:30–10:00  Samuel ROTH  
_On Lipschitz Constants and Entropy_

10:00–10:30  Coffee break

10:30–11:00  Zuzana ROTH  
_On the weakest version of distributional chaos - conjugacy problem_

11:00–11:15  Jakub ŠOTOLA  
_On the construction and differentiability of minimal non-invertible skew-product maps of 2-manifolds_


Thursday, 15 September

Morning session

Chair: Peter Raith

9:00–10:00  Maik GRÖGER  
_Mean equicontinuity and amorphic complexity_

10:00–10:30  Coffee break

10:30–11:10  Michal KUPSA  
_On the partitions with Sturmian-like refinements_
Friday, 16 September

Morning session

Chair: Dominik Kwietniak

9:00–9:30  Marta PIETRZYK
Quasi-uniform Convergence in Dynamical Systems Generated by Amenable Group Actions

9:30–10:00 Martha ŁĄCKA
Complexity Function for Pseudogroups

10:00–10:30 Coffee break

10:30–11:15 Peter RAITH
Transitivity for monotonic mod one transformations with two monotonic pieces
Abstracts

Presenting authors of joint projects are marked by *. 

On the structure of forbidden sets of rational difference equations

Francisco BALIBREA

Consider the difference equation of order $k$ given by the formula

$$x_{n+1} = f(x_n, x_{n-1}, ..., x_{n-k+1})$$

where $f : \mathbb{K}^k \to \mathbb{C}$ is of the form

$$f(x_n, ..., x_{n-k+1}) = \frac{P(x_n, x_{n-1}, ..., x_{n-k+1})}{Q(x_n, x_{n-1}, ..., x_{n-k+1})}$$

which is a rational difference equation (RDE) where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. The above equation can be seen as the following discrete dynamical system associate to the iteration function

$$F(x_n, x_{n-1}, ..., x_{n-k+1}) = (f(x_n, x_{n-1}, ..., x_{n-k+1}), x_n, ..., x_{n-k+2})$$

A solution of the equation is the sequence of numbers $(x_n)_{n=0}^{\infty}$ where $(x_0, ..., x_{k-1}) \in \mathbb{K}^k$ means a given vector of initial conditions.

An initial point $x_0$ is a forbidden point of an equation, if when constructing the corresponding solution, it has only a finite number of terms until $x_N$, because the next term $x_{N+1}$ is not defined.

The forbidden set of the equation, is the set of its forbidden points. Having forbidden sets is common in rational difference equations

$$x_{n+1} = R(x_1, x_2, ..., x_n) = \frac{P(x_1, x_2, ..., x_n)}{Q(x_1, x_2, ..., x_n)}$$

where $P(\cdot)$ is a polynomial of degree $n$ in its arguments and $Q(\cdot)$ is another polynomial of degree $m$.

The main aim of this talk is to construct explicitly forbidden sets for some equations, study their topological structure and state the same problem in the non-autonomous setting.
Mean equicontinuity and amorphic complexity

Maik GRÖGER

We investigate the relations of two complexity notions in the zero entropy regime: mean equicontinuity and amorphic complexity. As it turns out, there is a close relationship in the minimal setting and we will present further results highlighting the interplay of these two concepts. Further, for mean equicontinuous subshifts we prove that amorphic complexity corresponds to the box dimension of the maximal equicontinuous factor and for certain Toeplitz subshifts we show how to calculate amorphic complexity using the theory of iterated function systems. If time permits, we will also elaborate on possible extensions of these notions, in particular with respect to more general group actions. This is work in progress with Gabriel Fuhrmann, Tobias Jäger and Dominik Kwietniak.

Random dynamical systems generated by two Allee maps

Jozef KOVÁČ

In this talk, we will focus on random dynamical systems generated by two Allee maps. Two models are considered - with and without small random perturbations. We will show, that if we use strictly increasing Allee maps, than the behaviour of the systems is very similar to the behaviour of the non-stochastic system. However, in the case of unimodal Allee maps, the behaviour can dramatically change irrespective of the initial conditions.

Further results on the dynamics of spacing shifts

Marcin KULCZYCKI

We will recall the definition of a spacing subshift and the reason for the introduction of this class of maps. We will briefly go over the results regarding them that have been published so far and present answers to three open problems related to them: the question of their topological conjugacy, topological transitivity, and relative mixing.
On mixing completely scrambled systems

Jiří KUPKA

We show that there is a topologically mixing map which is completely scrambled. We also show that for any integer \( n \geq 1 \) there exists a continuum of topological dimension \( n \) supporting a transitive completely scrambled homeomorphism and an \( n \)-dimensional compactum supporting a weakly mixing completely scrambled homeomorphism. This solves a 15 years old open problem.

On the partitions with Sturmian-like refinements

Michal KUPSA

In this talk, we present some results about the evolution of partitions of a circle under the rotation.

In the dynamics of a rotation of the unit circle by an irrational angle \( \alpha \in (0, 1) \), we study the evolution of partitions whose elements are finite unions of left-closed right-open intervals with endpoints lying on the past trajectory of the point 0. We show that the refinements of these partitions eventually coincide with the refinements of a preimage of the standard “Sturmian” partition, which consists of two intervals \([0, 1 - \alpha)\) and \([1 - \alpha, 1)\). It means that even though we start with the partition that divides the circle into disconnected sets, its dynamical refinements eventually consist of intervals, i.e. connected sets. We reformulate this result in terms of Sturmian subshifts: we show that for every non-trivial factor mapping from a one-sided Sturmian subshift, satisfying a mild technical assumption, the sliding block code of sufficiently large length induced by the mapping is injective.

If time permits, some consequences for the return times statistics will be presented.

This is a joint work with Štěpán Starosta.

Complexity Function for Pseudogroups

Martha Łącka

During the talk we will examine the connections between complexity of a pseudogroup, its equicontinuity, the mixing property and entropy. We will prove that the entropy of a pseudogroup can be (under some additional assumptions) computed using a continuous and dynamically generating pseudometric. The talk is based on the joint work with Dominik Kwietniak.
Chaotic sub-dynamics in coupled logistic maps

Marek LAMPART* and Piotr OPROCHA

We study the dynamics of Laplacian-type coupling induced by logistic family $f_\mu(x) = \mu x(1-x)$, where $\mu \in [0, 4]$, on a periodic lattice, that is the dynamics of maps of the form

$$F(x, y) = ((1 - \varepsilon)f_\mu(x) + \varepsilon f_\mu(y), (1 - \varepsilon)f_\mu(y) + \varepsilon f_\mu(x))$$

where $\varepsilon > 0$ determines strength of coupling. Our main objective is to analyze the structure of attractors in such systems and especially detect invariant regions with nontrivial dynamics outside the diagonal. In analytical way, we detect some regions of parameters for which a horseshoe is present; and using simulations global attractors and invariant sets are depicted.

Periodic points of Lotka-Volterra map and their relation to number theory

Peter MALIČKÝ

Given the plane triangle

$$D = \{[x, y] : 0 \leq x, 0 \leq y, x + y \leq 4\}$$

we consider the map

$$F : D \to D, [x, y] \mapsto [x(4-x-y), xy]$$

and its periodic points. It is easy to show that a point $P = [x, 0] \in D$ is a fixed point of the map $F^n$ if and only if $x = 4 \sin^2 \frac{kn}{2^n \pm 1}$, where $n \geq 1$ and $k \geq 0$ are integers with $2k < 2^n \pm 1$. We are interested in interior periodic points of the map $F$. It is easy to verify that the point $[1, 2]$ is a fixed point of the map $F$. In [2] it was found the point $\left[1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right]$ with period 4. We have discovered the point $\left[1, \frac{3 + \sqrt{5}}{2}\right]$ with period 6. Our main result of [3] is a relation between lower and interior periodic points. Namely, if a point $[4 \sin^2 \frac{kn}{2^n \pm 1}, 0]$ is a saddle point of the map $F^n$ then there is an interior periodic point with the same itinerary with respect to the sets

$$A = \{[x, y] : 0 \leq x < 2, 0 \leq y \leq 4 - x\}$$

and

$$B = \{[x, y] : 2 < x \leq 4, 0 \leq y \leq 4 - x\}.$$  

From this fact it easy to derive the existence of many interior periodic points of he map $F$. The aim of our talk is to show that properties of periodic points of the map $F$ are closely related to some open problems in theory of numbers. These open problems are Artin’s conjecture on primitive roots, Wieferich primes and Sophie Germain primes.
References


Quasi-uniform Convergence in Dynamical Systems Generated by Amenable Group Actions

Marta PIETRZYK

We study properties of the Weyl pseudometric in a dynamical system generated by an amenable group action. Specifically we obtain several results regarding the interaction of the resulting topology with the entropy function and other important objects. These generalize the work of Downarowicz and Iwanik. As a corollary we obtain an alternative proof of Krieger’s theorem.

On dynamics of triangular maps of the square with zero topological entropy

Vojtěch PRAVEC

It is known that, for interval maps, zero topological entropy is equivalent to bounded topological sequence entropy as well as to the non-existence of Li-Yorke scrambled triples. In this paper we answer the question how the situation changes when instead of interval maps triangular maps of the unit square are concerned.
Transitivity for monotonic mod one transformations with two monotonic pieces

Peter RAITH

Suppose that \( f : [0, 1] \to [0, 2] \) is a strictly increasing continuous function. Define \( T_f x := f(x) \pmod{1} \). Then one calls \( T_f : [0, 1] \to [0, 1] \) a monotonic mod one transformation with two monotonic pieces. Furthermore assume that \( f \) is piecewise differentiable. Note that the condition \( \inf f' \geq \beta \) used later can be replaced by the weaker condition that \( |f(x) - f(y)| \geq \beta|x - y| \) holds for all \( x, y \in [0, 1] \).

If \( \inf f' \geq \sqrt{2} \) then the map \( T_f \) is topologically transitive. For any \( \beta < \sqrt{2} \) there is a function \( f \) with \( \inf f' \geq \beta \) such that \( T_f \) is not topologically transitive. Next assume that \( \inf f' \geq \sqrt{2} \) and \( f(0) \geq \frac{1}{\beta + 1} \). Then the map \( T_f \) is topologically transitive. However, for every \( \beta < \sqrt{2} \) there exists an \( f \) with \( \inf f' \geq \beta \) and \( f(0) \geq \frac{1}{\beta + 1} \) such that \( T_f \) is not topologically transitive. Moreover, if \( \beta \geq \sqrt{2} \) is sufficiently close to \( \sqrt{2} \) and \( \alpha < \frac{1}{\beta + 1} \) one can find a function \( f \) with \( \inf f' \geq \beta \) and \( f(0) \geq \alpha \) such that \( T_f \) is not topologically transitive. Nonetheless, in almost all cases where \( T_f \) is topologically transitive, it is also topologically mixing.

On Lipschitz Constants and Entropy

Samuel ROTH
(joint work with Jozef Bobok)

How can we interpret the infimum of Lipschitz constants in a conjugacy class of interval maps? For positive entropy maps, the exponential of the topological entropy gives a well-known lower bound. We show that for piecewise monotone maps, these two quantities are equal, but for countably piecewise monotone maps, the inequality can be strict. Moreover, in the transitive and Markov case, we characterize the infimum of Lipschitz constants as the exponential of the Salama entropy of a certain reverse Markov chain associated with the map. Dynamically, this number represents the exponential growth rate of the number of iterated preimages of nearly any point.

On the weakest version of distributional chaos - conjugacy problem

Zuzana ROTH
(joint work with Jana Hantáková and Samuel Roth)
The conjugacy problem for DC3 chaos was first studied 11 years ago. It was claimed that the existence of a DC3 pair is not conjugacy invariant; unfortunately the example given (in 2005) overlooked an uncountable DC3 set. In our work we show a correct example where the existence of a DC3 pair is destroyed by conjugacy. We also improve this result for the existence of an uncountable DC3 set.

Loops of transitive interval maps

Lubomír SNOHA  
(joint work with Sergii Kolyada and Michał Misiurewicz)

We continue investigation of the topology of the spaces of transitive interval maps. We show that loops that are not contractible in some of those spaces, can be contracted in slightly larger spaces. We also describe the topology of the space of unimodal and bimodal transitive maps with constant slope.

On the construction and differentiability of minimal non-invertible skew-product maps of 2-manifolds

Jakub ŠOTOLA  
(joint work with Sergei Trofimchuk)

In [5] the authors constructed a non-invertible minimal map of a torus, in [1] it was proven that the only 2-dimensional manifolds admitting minimal maps are tori and Klein bottles and their unions. Examples of such homeomorphisms are known since 1960’s (e.g. [3] or [6]). On the other hand, the question whether there exists a non-invertible minimal map of a Klein bottle remained open until recently, [7].

The construction of such mapping (using methods from [4]) is the cornerstone of this talk. Also it is known due to [2] that there is no analytic minimal non-invertible map on a 2-manifold. Hence the differentiability and Lipschitz continuity of minimal non-invertible maps of 2-manifolds will be discussed as well.

References


On generic and dense chaos for maps induced on hyperspaces

Michaela MLÍCHOVÁ and Marta ŠTEFÁNKOVÁ*

A continuous map $f$ on a compact metric space $X$ induces in a natural way the map $\tilde{f}$ on the hyperspace $\mathcal{K}(X)$ of all closed non-empty subsets of $X$. We study the question of transmission of chaos between $f$ and $\tilde{f}$. We deal with generic, generic $\varepsilon$-, dense and dense $\varepsilon$-chaos for interval maps. We prove that all four types of chaos transmit from $f$ to $\tilde{f}$, while the converse transmission from $\tilde{f}$ to $f$ is true for generic, generic $\varepsilon$- and dense $\varepsilon$-chaos. Moreover, the transmission of dense $\varepsilon$- and generic $\varepsilon$-chaos from $\tilde{f}$ to $f$ is true for maps on general compact metric spaces.

Generic chaos on graphs

Michal TAKÁCS

We study generically chaotic continuous maps on graphs. We show that generic chaos is equivalent to generic $\varepsilon$-chaos for some $\varepsilon > 0$. This in particular implies that generic chaos on graphs can be described in terms of behaviour of open sets under the iterates of the map. We also give a characterization of generically chaotic maps in terms of topologically transitive subgraphs.
List of participants

Francisco BALIBREA  
*University of Murcia, Spain*  
balibrea@um.es

Jernej ČINČ  
*University of Vienna, Austria*  
jernej.cinc@univie.ac.at

Gabriel FUHRMANN  
*Friedrich Schiller University Jena, Germany*  
gabrielfuhrmann@googlemail.com

Maik GRÖGER  
*Friedrich Schiller University Jena, Germany*  
maik.groeger@uni-jena.de

Jana HANTÁKOVÁ  
*Silesian University in Opava, Czech Republic*  
jana.hantakova@math.slu.cz

Zdeněk KOČAN  
*Silesian University in Opava, Czech Republic*  
zdenek.kocan@math.slu.cz

Jozef KOVÁČ  
*Comenius University in Bratislava, Slovakia*  
jozefkovac314@gmail.com

Marcin KULCZYCKI  
*Jagiellonian University in Kraków, Poland*  
Marcin.Kulczycki@im.uj.edu.pl

Jiří KUPKA  
*University of Ostrava, Czech Republic*  
jiri.kupka@osu.cz

Michal KUPSA  
*Academy of Sciences of the Czech Republic, Czech Technical University in Prague, Czech Republic*  
kupsa@utia.cas.cz

Dominik KWIEȚNIAK  
*Jagiellonian University in Kraków, Poland*  
Dominik.Kwietniak@im.uj.edu.pl

Marta ŁACKA  
*Jagiellonian University in Kraków, Poland*  
marta.ubik@gmail.com

Marek LAMPART  
*VSB - Technical University of Ostrava, Czech Republic*  
marek.lampart@vsb.cz

Peter MALIČKÝ  
*Matej Bel University, Slovakia*  
Peter.Malicky@umb.sk

Marta PIETRZYK  
*Jagiellonian University in Kraków, Poland*  
martap89@gmail.com

Vojtěch PRAVEC  
*Silesian University in Opava, Czech Republic*  
vojtech.pravec@math.slu.cz

Peter RAITH  
*University of Vienna, Austria*  
Peter.Raith@univie.ac.at

Samuel ROTH  
*Silesian University in Opava, Czech Republic*  
samuel.roth@math.slu.cz
Zuzana Roth
Silesian University in Opava, Czech Republic
zuzana.roth@math.slu.cz

Lenka Rucká
Silesian University in Opava, Czech Republic
lenka.rucka@math.slu.cz

Jaroslav Smital
Silesian University in Opava, Czech Republic
jaroslav.smital@math.slu.cz

Lubomír Snoha
Matej Bel University, Slovakia
Lubomir.Snoha@umb.sk

Jakub Sotola
Silesian University in Opava, Czech Republic
jakub.sotola@math.slu.cz

Marta Štefánková
Silesian University in Opava, Czech Republic
marta.stefankova@math.slu.cz

Michal Takács
Matej Bel University, Slovakia
michal.takacs@umb.sk

Jan Tesarčík
Silesian University in Opava, Czech Republic
jan.tesarck@math.slu.cz

Benjamin Vejnar
Charles University, Czech Republic
benvej@gmail.com